Lab Report

Introduction

What is spring oscillation?

Spring oscillation is a form of simple harmonic motion, otherwise known as, periodic sinusoidal oscillation. Simple harmonic motion is found in light waves, sound waves, springs and pendulums, and thus is a fundamental form of motion within physics (Khan Academy, 2018). Springs exert a restorative force, in the sense that the spring is always attempting to restore itself to equilibrium (Better Explained, 2018). In order to achieve equilibrium, it must exert a directly proportionate force to the one acting upon it. This is observed in the formula for Hooke's Law stating that $f = kx$ (Britannica, 2018 ; Osbourne, 2018). In this *f* is the amount of both external force acting upon the spring as well as the restorative force of the spring itself. *F* is proportionate and dependant upon *x*, the displacement of the spring as well as *k*, the spring constant. The spring constant is defined as the rigidity of the spring, or the amount of work required to displace it. Higher spring constants, measured in newtons per metre (n/m) exert a larger amount of force per given distance and thus require an equal force to displace (Britannica, 2018 ; Osbourne, 2018).

How does a spring oscillate?

The oscillation of a spring follows this process:

1. Equilibrium - at this point the spring is perfectly at rest, no forces are acting upon it.
2. The introduction of a force, and initial stretch to full positive displacement - at this point the spring has reached maximum displacement in a positive direction. The kinetic energy of the spring is zero but here are large amounts of potential energy.
3. Pass through equilibrium - given that the spring holds elastic and restorative force, it will exert an equal amount of force to pull the spring away from maximum positive displacement. In doing so the spring will pass through its original equilibrium position, and due to the momentum developed, will reach its maximum velocity.
4. Maximum negative displacement - after passing through its equilibrium position, the spring will expand until its maximum negative displacement before the lower part of the spring exerts force and pulls it back through equilibrium.
5. Repetition - the spring will oscillate back and forth between equilibrium, positive and negative displacement until all kinetic energy is released.

(Khan Academy, 2018, Indiana University, 1999; physics.net, 2018)

Why does a spring oscillate? How is the nature of the spring and its energy changing?

The law of conservation of energy states that energy cannot be created or destroyed; it can only change forms (SparkNotes, 2018; Lumen Learning, 2018). In most reactions, the change of forms happens through friction or a exothermic reaction. In the case of an ideal spring, where friction isn’t considered, the shift in energy occurs through motion, specifically oscillation. As described above, when a spring reaches its maximum positive displacement there is a

substantial supply of potential energy (SparkNotes, 2018; Lumen Learning, 2018). To restore equilibrium, the spring must first release this energy. The energy is transferred from potential to kinetic in the form of oscillation. The formula for determining the potential energy held by a spring is $U_s = \frac{1}{2}kx^2$, meaning that the potential energy is equal to half of the force (kx) squared. The potential energy of a spring is most notable at full compression and expansion (SparkNotes,2018). The kinetic energy is calculated by observing the formula $KE_{max} = \frac{1}{2}mv^2 = \frac{1}{2}kx_{max}^2$. This formula displays that the maximum kinetic energy is proportional to half of the mass, multiplied by the velocity squared. This is also equal to half of the spring constant multiplied by the displacement squared. Comparatively, we see that both potential and kinetic energy are equal to $\frac{1}{2}kx^2$. Proving that kinetic and potential energy are proportionate to one another. This equation can also be used to solve for the maximum velocity $v_{max} = x_{max}\sqrt{\frac{m}{k}}$. This equation states that the maximum velocity is dependant upon the maximum displacement, mass and spring constant. The sum of PE (potential energy) and KE (kinetic energy) results in the mechanical energy of a given spring.

Aim

The aim of this experiment is to measure the period of oscillation, and maximum displacement of springs with varying spring constants, as well as to observe the relationship between applied force, displacement and oscillation period.

Hypothesis

The spring with a higher constant will require a greater force to increase its displacement. As all springs are being measured at a constant force, the higher spring constant will result in the lowest displacement. The combination of lowered displacement and an increased acceleration due to the larger restorative force of a high spring constant will result in a shorter period of oscillation. The higher spring constant exerts a larger amount of force (newtons) per metre resulting in the aforementioned acceleration. The displacement of the spring is directly proportional to it's spring constant, as observed in a variation of the Hooke's Law formula $x = \frac{f}{k}$ .These two relationships show that period, displacement and spring constant are all somewhat proportionate and dependant upon one another.

Variables

Dependant	Independant	Constant
• Positive displacement	• Spring constant	• Applied force (m x g)
• Oscillation period		

Materials

- 1 x stopwatch
- 1 x spring clamp
- 1 x clamp stand
- 3 x springs, with varying constants
- 1 x Ruler

Method

Hooke's Law experiment procedure:

1. Tighten spring clamp firmly around clamp stand.
2. Place spring on end of clamp.
3. Measure the length of the spring at equilibrium.
4. Attach a weight to the end of spring.
5. Measure the length of the spring at full positive displacement.
6. Calculate the force (m x g).
7. Calculate the displacement by subtracting the length of the spring at equilibrium from its length at full displacement.
8. Observe the formula $f = kx$ and solve for k ($k = \frac{f}{x}$).
9. Record k (spring constant) as newtons per meter.

Spring oscillation experiment procedure:

1. Complete Hooke's Law experiment and determine springs with three different spring constants.
2. Attach first spring to the clamp.
3. Measure the length of each spring at equilibrium.
4. Attach weight of 0.3 kg to a spring.
5. Start stopwatch and wait until spring reaches full displacement and returns to equilibrium before stopPing.
6. Mark length of full displacement.
7. Repeat experiment three times for each spring.

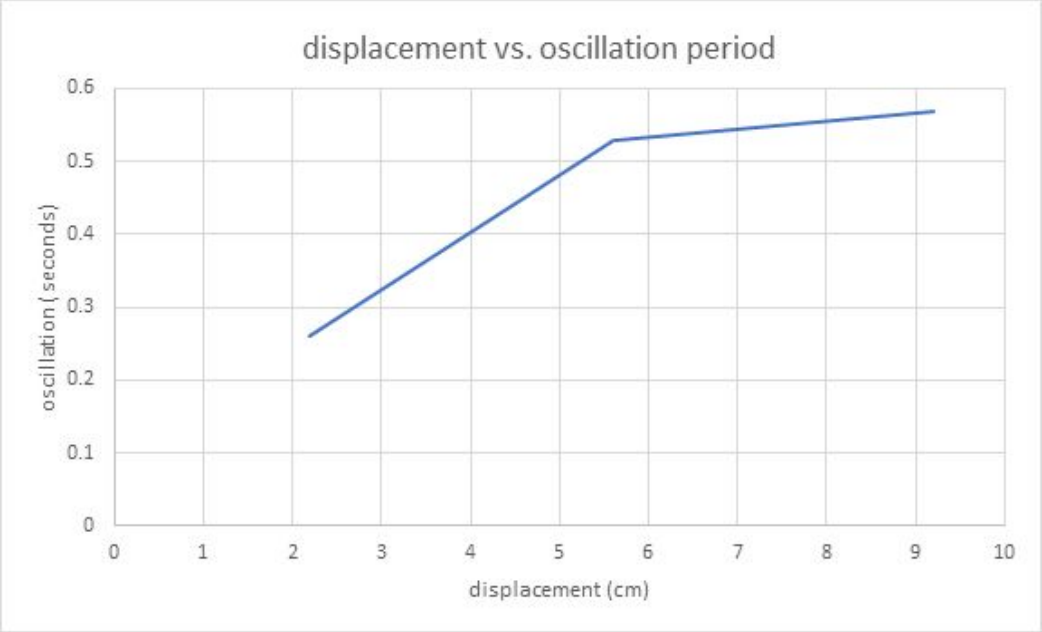
Results

Raw data:

Spring type	Spring a (k = 30.9)		Spring B (k = 49)		Spring C (k = 147)	
Test number	Period	Displacement	Period	Displacement	Period	Displacement
Test 1	00.50 s	0.093 m	00.44 s	0.056 m	00.34 s	0.024 m
Test 2	00.62 s	0.092 m	00.63 s	0.057 m	00.37 s	0.023 m
Test 3	00.60	0.092 m	00:54 s	0.055 m	00.38 s	0.021 m

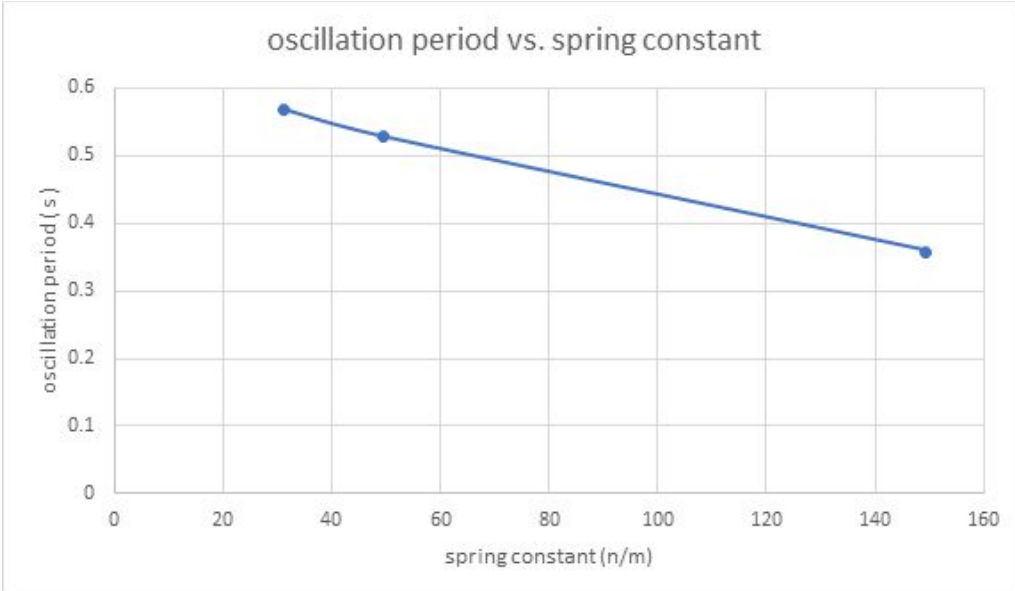
Averages:

Spring constant	Average period	Average displacement
Spring A (k = 30.9)	0.57 s	0.092 m
Spring B (k = 49)	0.53 s	0.056 m
Spring C (k = 147)	0.36 s	0.022 m



(figure 1)

(figure 2)



(figure 3)

Analysis

The results show a clear correlation between the spring constant, the maximum positive displacement and the spring's period of oscillation. Figure one represents the relationship between the spring constant and the displacement. The graph observes that a higher spring constant will result in a lower displacement. As the results were measured under a constant force of 2.8 newtons, the lesser displacement is due to the fact that a greater spring constant requires more force, or work, to stretch, and thus the lower spring constants yield a greater displacement.

Figure two demonstrates that a greater displacement leads to a slower period of oscillation. As observed in figure one, lower spring constants induce larger amounts of displacement, thus a lower spring constant will have a slower period of oscillation. This exact relationship is observed in figure three in which the oscillation period is shown to be somewhat proportional to the spring constant. This relationship is due to the greater amount of force exerted per metre by higher spring constants, resulting in a greater acceleration.

Theoretical formula vs actual results:

As with all physics, results of a given experiment can be calculated and predicted using scientific formulas. In the instance of the spring oscillation experiment two factors can be deduced using the Hooke's Law equation, $f = kx$. The spring constant can be determined by solving for k as such: $k = \frac{f}{x}$. Then the displacement can be calculated by solving for x, $x = \frac{f}{k}$. The period of oscillation (T), however, must be calculated by observing the formula $T = 2\pi\sqrt{\frac{m}{k}}$.

In which the period of oscillation (T) is proportional to two Pi multiplied by the root of mass divided by spring constant. Conceptually, this calculation displays the dependence of period on force and spring constant, proving the hypothesis. It also represents the relationship between oscillation, or general simple harmonic motion and sine, as Pi is present within the equation.

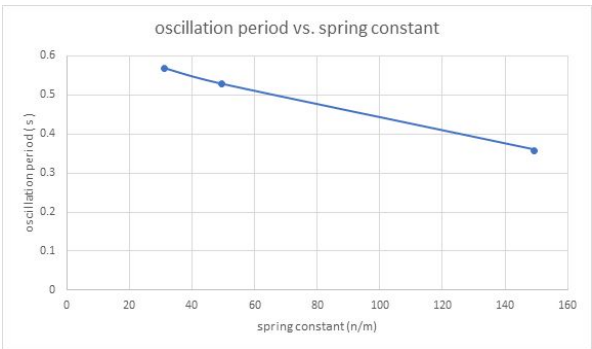
According to the formula these were the expected results compared with the ones recorded during the experiment:

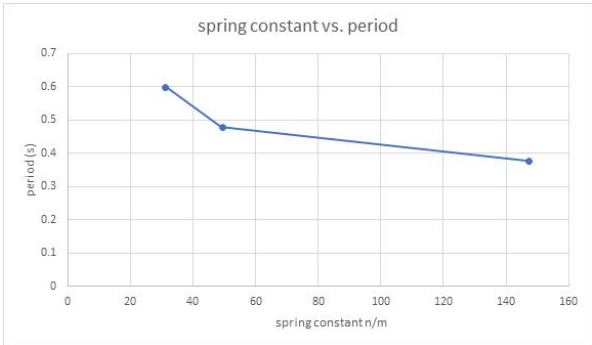
Spring constant	Theoretical period	Actual period (average)	Theoretical displacement	Actual displacement (average)
Spring A (k = 30.9)	00.11 s	00.57 s	0.095 m	0.092 m
Spring B (k = 49)	00.07 s	00.53 s	0.056 m	0.06 m
Spring C (k = 147)	00.02s	00.36 s	0.019 m	0.022 m
Average percentage error	1037.96% error		8.61% error	

The above figure displays a large percentage error regarding oscillation period, proving that the results were significantly incorrect. The high percentage error is due to the delayed reaction time and sensitivity of the stopwatch, paired with the ill-considered decision to record only *one* full oscillation. The speed of one oscillation is too great to be measured accurately. This analysis paved way for a separate experiment, designed to accurately re-calculate the oscillation period. The displacement measurements remained constant to the previous experiment, however, a total of ten full oscillations were recorded, rather than one. Once the speed of ten oscillations was measured it was necessary to divide the measurement by ten to deduce the period of oscillation for each spring constant. In the previous experiment, a trend of lengthened displacement was noticed, most likely due to distortion form excess mass, and thus to rectify this the applied mass was diminished from 0.3 kg to 0.25 kg. While these factors may be different the overall trend and correlation between spring constant, applied force, displacement and period should remain.

Spring constant	Theoretical period	Actual period	Average percentage error
Spring A (k = 30.9)	0.101 s	0.602 s	451 % error
Spring B (k = 49)	0.064s	0.481 s	
Spring C (k = 147)	0.021 s	0.379 s	

The second experiment yielded results that were not affected by spring distortion, which lessens the springs tension or constant and thus affects the oscillation period. While these results had a high percentage error, likely still due to reaction time and perhaps confusion regarding keePing track of the oscillations, the accuracy increased by 586.96%. If further testing were to be conducted, a larger group would be involved, one individual per measurement as well as a more accurate stopwatch. Perhaps the second experiment could have been conducted under the same force of 2.8 newtons, rather than the second experiment, in which a force of 2.45 newtons was introduced. Had the force remained, the culmination of results would have been more constant and cohesive. That being said, the hypothesis is still proven regardless - a higher spring constant will result in a lower oscillation period.





A comparison of the graphs show an almost identical trend, further proving that the spring constant has a relationship with the period of oscillation. The graph moves in a non-linear negative slope, showcasing that as the spring constant increases the period of oscillation decreases, therefore spring constant and oscillation period have an inverse relationship.

Frequency

To further analyse the results of the experiment, it is possible to deduce the frequency of oscillation. As mentioned, all harmonic motion admits a frequency, or the amount of cycles/ repetitions, or in this case oscillations that occur within a given time frame (Lumen Learning, 2019). When analysing frequency it is vital to know that period and frequency are reciprocals and thus, a smaller period will result in a higher frequency. The formula to observe frequency of oscillation is $f = \frac{1}{T}$. Meaning that the frequency is equal to the quotient of one second and the oscillation period, meaning that frequency records the number of cycles, or in this case oscillations, that occur in one second. (Physics.net, 2018). Based on the results of the second experiment, recording a more accurate oscillation period, the frequency of each spring constant is as follows:

Spring constant	Oscillation period	Frequency
30.9 n/m	0.602 s	1.66 Hz
49 n/m	0.481 s	2.07 Hz

147 n/m	0. 379 s	2.63 Hz

Conclusion

Overall, the results coincide with the hypothesis. A clear relationship between all factors was proven, as was the fact that period and spring constant are directly connected, as are spring constant and displacement and thus, displacement and period. It remains unclear whether the relationship between displacement and period is causation or correlation. Perhaps their similarities are apparent because both quantities are directly proportionate and dependant to the spring constant.

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